Probing e^+e^- annihilation in noncommutative electroweak model

Chien Yu Chen[1, *]

[1]*Department of Physics, National Tsing-Hua University, Hsinchu 300, Taiwan*

If the twist *Poincaré* transformation is imposed on the noncommutative spacetime, then *Lorentz* invariance cannot be applied on QFT. To data, noncommutative theory is one of the best candidates to modify *Lorentz* transformation. In this paper, we argue parity violation under the process of $e^+e^- \rightarrow \gamma\gamma$ and make a detailed analysis of the difference behavior of each helicity state on noncommutative spacetime. The effect arises from the production of spin and magnetic fields. We check the energy momentum conservation for all used couplings and discover that if the electric field changes particle energy spectrum, there is no symmetry violation as the field produces a longitudinal state on the finial triple boson couplings.

PACS numbers: 11.10.Nx, 12.60.Cn, 13.88.+e, 11.30.Er

I. INTRODUCTION

Lorentz symmetry constrains the transformation of spacetime from boost and rotation. Many phenomena that cannot be predicted by the standard model are expected to violate *Lorentz* symmetry. The main purpose of this paper is to concentrate on the effects induced by the background magnetic field. In particular, we discuss the parity asymmetry in $e^+e^- \rightarrow \gamma\gamma$ with the content of *Lorentz* violation. The numerical results present that parity is violated, while CP symmetry still preserves in the next leading $\theta_{\mu\nu}$ patch with noncommutative background.

Furthermore, CP symmetry puts a constraint on the cross section. Parity violated phenomena simultaneously violates the charge conservation. This effect induces a slight space translation, but charge violated event changes the magnitude in total cross section. In the viewpoint of quantum gravity, energy scale of *Lorentz* violation is ranged in the Plank scale, $M_{PL} = 10^{19}\ GeV$. In this paper, we probe the effects of backgound field direction on the total cross section. The scale Λ_C and the colliding energy level have been set to 1 TeV and 800 GeV respectively. The total cross section fluctuation associates with $70K_{Z\gamma\gamma}\cos\alpha_B$ (fb) in summing each photon polarization, where $K_{Z\gamma\gamma}$ is a triple gauge boson coupling, and α_B is the direction of background magnetic field. The shift is miniscule in comparing with the standard model cross section 5560 (fb).

Moreover, due to the interaction between background magnetic field and photons with oppsite polarization, the spin-magnetic interaction, $\vec{S}\cdot\vec{B}$, produces a forward-backward asymmetry cannot be predicted by the standard model. In calculations, the sensitive phenomenon of the total cross section of central energy is pertaining to the direction of background magnetic field. We observe that the spin-magnetic interaction effect is changed under the relation, $<\vec{S}>\cdot\vec{B} = \pm|B|\cos\theta_{B_Z}$. The principal frame takes along z-axis, and the total energy spectrum is proportional to the θ_{B_Z} angle. By the way, the electric field is absent due to the unitarity constraint.

Most investigations tend to choose a preferred direction of the isotropic and homogeneous earlier universe, i.e. by adding a nonlocal four vector term in the Lagrange [1]. The field along the direction of the background field equally imposes a constant direction rearranging the order of spacetime [2]. There are some papers consider noncommutative scalar field in $fuzzy$ sphere [3], this spacetime considers the era of universe earlier than cosmology scale. However, many theories with consistent concepts are to define a preferred direction on the isotropic spacetime. This is apparently to oppose the general assumption of *Lorentz* symmetry. Noncommutative field theory is one of the theories violates *Lorentz* symmetry in putting a constant background field term in the $Dirac\ Born\ Infeld$ action of the bosonic string [1]. The field influences the position between particles cannot be exchanged on the same consequence.

In the concept of noncommutive spacetime, there are three kinds of structure [4] considered: (1) canonical structure, (2) Lie algebra structure, and (3) quantum space structure. It dominates to decide a way by $\star$ production. We cannot think of a different description of the gauge transformation into a different map. On the model building, unfortunately, the noncommutative model has been restricted by the *No-Go Theorem* [5]. Only $U_\star(1)$ gauge group can build into this spacetime. Separating the generator into U(1) gauge and $SU_\star(N)$ parts, the redefined relation between each particle can be formed a group to aside a existence of condensed field. Requiring non-abelian representation in considering enveloping algebra [4, 6], it separates the generators of different commutation relations and expands the gauge representation of infinite $\theta_{\mu\nu}$ deformation. Renormalization implies unitary constraint is satisfied in the field theory and restricts $\theta_{\mu\nu}$ is just considered into first order.

The UV/IR mixing commits the particle to propagate nonlocal space. The mixed angular momentum from background constant direction and particle kinetic momentum towards to renew the commutation form or split the $U(1)$ generation. Due to slip the photon polarization under modifying commutation relation will

*Electronic address: d9522817@phys.nthu.edu.tw

absorb some physical degrees of freedom into the lost generators, the complete physical field is still in the unbroken gauge. Hereafter, the redefined angular momentum is considerable to modify $U(1)$ gauge. We do not take the condensed picture in gauge boson, the couplings used concentrate on the unbroken $U(1)$ generators with preserving chiral symmetry in fermion fields. Hence, the behavior of finial photon polarization is the simple consequence of background nonlocal vector condensation with particle polarizations.

Expanding the origin nonabelian gauge theory in noncommutative spacetime and considering the enveloping algebra modifies gauge representation. Using *Seiberg Witten* map [1, 7], the origin gauge group of the standard model is extended by first order $\theta_{\mu\nu}$ deformation under noncommutative phase-like translation,

$$f(x)\star g(x) = f(x)exp\left(\frac{i}{2}\overleftarrow{\partial_\mu}\theta^{\mu\nu}\overrightarrow{\partial_\mu}\right)g(x).$$

Which are two function products of $\star$ deformed noncommutative algebra. The gauge group is extended as $SU_{\star C}(3)\otimes SU_{\star L}(\mathrm{L})\otimes U_{\star Y}(1)$ with produced background deformation by preserving gauge restriction. In the next section, we briefly introduce gauge boson action using enveloping algebra expansion [4]. Thereof, all of the field theory involving $\theta_{\mu\nu}$ deformation comments the physics ordered phase, and contains the information of earlier universe background magnetic and electric field.

Using the properties of space and momentum exchange under Moyal space, the *Lorentz* group SO(1,3) is isomorphic to O(1,1)$\otimes$SO(2), where the lost generators are residing in the hypersurface. It results from an arbitrary generator and uniquely choose in the background field direction and violates boost and rotation symmetry. These phenomena induce parity symmetry violated effects. The common commutation relation on the four vector spacetime is

$$[x_\mu, x_\nu]_\star = i\theta_{\mu\nu} = i\frac{C_{\mu\nu}}{\Lambda^2_{NC}}, \tag{1}$$

and

$$C_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix}, \tag{2}$$

where $\theta_{\mu\nu}$ contains all the information of the background field, such as the field strength tensor of electrodynamics. The cross section is charge violated due to parity violation and CP conservation of odd order theta deformation.

However, the spin of any physics field interacts with background field in odd order $\theta_{\mu\nu}$ deformations. If we choose a preferred direction on the homogeneous and isotropic spacetime, under the background, parity does not remain a perfectly symmetry. Each particle helicity induces an opposite contribution on coupling to the background field. Particle energy spectrum is exchanged by the spin and background magnetic interaction. Therefore, if the deviation of each helicity dispersion is the same, the total parity violated phenomenon will be invisible. On the other hand, each photon helicity induces an opposite contribution on forward-backward asymmetry, the unpolarized electron initial beams will produce an asymmetric deviation to each helicity of photon luminosity.

II. BRIEF REVIEW OF NONCOMMUTATIVE THEORY

On the commutative spacetime we use the Seiberg-Witten map to generate noncommutative theta deformed potential. The series ordered $\theta_{\mu\nu}$ expansion in enveloping algebra extends the non-abelian gauge symmetry from $SU(2)\otimes U(1)$ to $SU_{\star L}(\mathrm{L})\otimes U_{\star Y}(1)$ [4, 6]. The standard noncommutative model is invariant under the gauge transformation builded by Hopf algebra [6, 7, 8] on Moyal space [9]. It supposes the existence of an infinitesimal transformation generator X with $\phi \longmapsto \mathrm{X}\triangleright\phi$. The action of the field is multiplied by a coproduct $\triangle$, denoted in $\phi\otimes\psi \longmapsto \triangle(\mathrm{X})\triangleright(\phi\otimes\psi)$.

The translation of coproduction between the twist deformation and the initial form,

$$\triangle_\theta(X) = \mathfrak{F}^{-1}\triangle_0(X)\mathfrak{F} = \mathfrak{F}^{-1}(X\otimes 1 + 1\otimes X)\mathfrak{F}, \tag{3}$$

and the noncommutative momentum translation representation,

$$\mathfrak{F} = exp(-\frac{i}{2}\theta^{ij}p_i\otimes p_j), \tag{4}$$

are defined by abelian gauge transformation. The coproduct of Poincar*é* generator requires a consistent deformation between two fields, $m_0(\phi\otimes\psi) = \phi\cdot\psi$, and isomorphic to $m_\theta(\phi\otimes\psi) = \phi\star\psi$. Therefore, the translation of the gauge symmetry under this rule is similarly to take Eq.(2.1) and Eq.(2.2) into

$$\begin{aligned} &X\triangleright m_0(\phi\otimes\psi) = m_0(\triangle_0(X)\triangleright(\phi\otimes\psi)) \\ &\longmapsto X\triangleright m_\theta(\phi\otimes\psi) = m_0(\triangle_\theta(X)\triangleright(\phi\otimes\psi)). \end{aligned}$$

We use this representation to prove photon polarization does not be changed in the noncommutative spacetime. However, if ψ and ϕ are substituted for four vector momentum, and $Pauli-Ljubanski$ polarization four vectors individually,

$$\mathbb{W}^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}J_{\nu\alpha}P_\beta, \tag{5}$$

in which $J_{\nu\alpha}$ is *Lorentz* rotation and boosts generator, P_β is the momentum operator.

Hence, after transformation it is easily shown that, for chargless particle, *Lorentz* tensor violates the origin translation and rotation in isotropic and homogeneous spacestime. On the other hand, if the field contains a charge, momentum translation is violated along the background electric field direction. Photon is a chargeless particle, the direction of translation will not induce another degree of freedom to generate its mass. In fact that noncommutative is translational invariance in Eq.(2.1, 2.2, 2.3). Following above discussion, m_θ ($P_\mu\otimes P_\nu$ - $P_\nu\otimes P_\mu$) = 0 takes a constraint

on $Pauli - Ljubanski$ polarization. The commutation relations $m_\theta(\mathbb{W}^\mu \otimes P^\nu$- $P^\nu \otimes \mathbb{W}^\mu) = 0$, $P^2 = m^2$ and $\mathbb{W}^2$ = m^2s(s+1) still retain the properties of $Casimir$ operator, where m is particle mass along to the direction of momentum and s is its polarization. For the massless case, $\mathbb{W}^2 = 0$, and m = 0, photon does not contain a longitudinal state even after momentum translation. Therefore, gauge condition $m_\theta(P^\mu \otimes \mathbb{W}_\mu) = 0$ is still unchanged. However, the summation of polarization should add a phase $\phi \sim \vec{B} \cdot (\vec{P}_1 \times \vec{P}_2)$ due to two gauge bosons product.

The noncommutative gauge theory is very interesting in which contains many degrees of freedom from choosing a different representations of gauge kinetic term under trace technique. On this way, we use the enveloping algebra to realize the nonabelian group[4], and choose a minimal expression of gauge expansion. By dividing the gauge kinetic term, one part is minimal and another is non-minimal. The gauge action of noncommutative electroweak model[10] is regarded as

$$S_{gauge} = S_{gauge}^{minimal} + S_{gauge}^{nm-term}, \tag{6}$$

the minimal term is to expand the origin using $Seiberg\ Witten$ map. In order to consider a triplet gauge boson couplings, hence, the non-minimal term is to choose a different trace technique on the aspect of gauge boson parameter to expand the gauge boson action,

$$\begin{aligned} &S_{gauge}^{minimum} = \\ &-\frac{1}{2}\int d^4x\left(\frac{1}{2}A_{\mu\nu}A^{\mu\nu} + TrB_{\mu\nu}B^{\mu\nu} + TrG_{\mu\nu}G^{\mu\nu}\right) \\ &+\frac{1}{4}g_s d^{abc}\theta^{\rho\sigma}\int d^4x\left(\frac{1}{4}G^a_{\rho\sigma}G^b_{\mu\nu} - G^a_{\rho\mu}G^b_{\sigma\nu}\right)G^{\mu\nu,c} + O(\theta^2), \end{aligned}$$

and

$$\begin{aligned} &S_{gauge}^{nm-term} = \\ &g'^3 k_1\theta^{\rho\sigma}\int d^4x\left(\frac{a}{4}A_{\rho\sigma}A_{\mu\nu} - A_{\mu\rho}A_{\nu\sigma}\right)A^{\mu\nu} \\ &+ g'g^2k_2\theta^{\rho\sigma}\int d^4x\left[\left(\frac{a}{4}A_{\rho\sigma}B^a_{\mu\nu} - A_{\mu\rho}B^a_{\nu\sigma}\right)B^{\mu\nu,a} + c.p.\right] \\ &+ g'g_s^2k_3\theta^{\rho\sigma}\int d^4x\left[\left(\frac{a}{4}A_{\rho\sigma}G^b_{\mu\nu} - A_{\mu\rho}G^b_{\nu\sigma}\right)G^{\mu\nu,b} + c.p.\right] \\ &+ O(\theta^2), \end{aligned}$$

the first one is the origin gauge boson kinetic term on noncommutative spacetime, the parameter "a" is an extra gauge degrees of freedom. In this paper, we set the constant parameter to 3 by imposing renormalization and unitary conditions. Another is non-minimal term, considering the freedom of different trace technique on kinetic gauge field to construct the non-minimal version of $mNCSM$ in using the different Seiberg-Witten map.

Each triple gauge boson coupling is derived from the above action. Extracting the couplings from the Lagrange, couplings of $\gamma-\gamma-\gamma$ and $Z-\gamma-\gamma$ are presented as follows,

$$\mathfrak{L}_{\gamma\gamma\gamma} = \frac{e}{4}\sin 2\theta_W K_{\gamma\gamma\gamma}\theta^{\rho\sigma}A^{\mu\nu}\left(aA_{\mu\nu}A_{\rho\sigma} - 4A_{\mu\rho}A_{\nu\sigma}\right) \tag{7}$$

$$\begin{aligned} \mathfrak{L}_{Z\gamma\gamma} = \frac{e}{4}\sin 2\theta_W K_{Z\gamma\gamma}\theta^{\rho\sigma}&\left[2Z^{\mu\nu}\left(2A_{\mu\rho}A_{\nu\sigma} - aA_{\mu\nu}A_{\rho\sigma}\right)\right. \\ &\left.+ 8Z_{\mu\rho}A^{\mu\nu}A_{\nu\sigma} - aZ_{\rho\sigma}A^{\mu\nu}A_{\mu\nu}\right], \end{aligned}$$

where the couplings $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ contain the gauge parameters, g, g_s, and g'. Ref.[11] plots the range of all these couplings and also makes more detailed analysis to give a constraint. The couplings are composed by g_i, i goes from 1 to 6. The C and P are violated in these couplings, but preserves CP symmetry in non-planar tree level diagram.

On the Seiberg-Witten map, there are some kinds of coupling induced by the connection of $\theta_{\mu\nu}$. Following the electroweak model [10], the change up to the first order $\theta_{\mu\nu}$ modification uses the enveloping algebra to extend the non-abelian gauge group. The interesting coupling Z-γ-γ violates the angular momentum distribution[12], hence it is exactly forbidden on the commutative standard model. Approximately, the branching radio of Z $\rightarrow \gamma\gamma$ is 4×10^{-8}, and the range of coupling are - 0.333< $K_{Z\gamma\gamma}$ <0.095 and - 0.184< $K_{\gamma\gamma\gamma}$ <- 0.419[11]. In this paper, we set $K_{Z\gamma\gamma}$ = - 0.2, and $K_{\gamma\gamma\gamma}$ = - 0.3 for convenient.

In renormalization aspects, the triple coupling tensor $\Theta^{\mu\nu\rho}$ is changed by choosing a different map [7, 13]. However, the map produces a geometric freedom in gauge sector. The triple gauge boson coupling tensor is

$$\begin{aligned} &\Theta_3^{\mu\nu\rho}(a; k_{\mu1}, k_{\nu2}, k_{\rho3}) = \\ &-(k_1\theta k_2)\left[(k_1-k_2)^\rho g^{\mu\nu} + (k_2-k_3)^\mu g^{\nu\rho} + (k_3-k_1)^\nu g^{\rho\mu}\right] \\ &-\theta^{\mu\nu}[k_1^\rho(k_2k_3) - k_2^\rho(k_1k_3)] - \theta^{\nu\rho}[k_2^\mu(k_3k_1) - k_3^\mu(k_2k_1)] \\ &-\theta^{\rho\mu}[k_3^\nu(k_1k_2) - k_1^\nu(k_3k_2)] \\ &+(\theta k_2)^\mu[g^{\nu\rho}k_3^2 - k_3^\nu k_3^\rho] + (\theta k_3)^\mu[g^{\nu\rho}k_2^2 - k_2^\nu k_2^\rho] \\ &+(\theta k_3)^\nu[g^{\mu\rho}k_1^2 - k_1^\mu k_1^\rho] + (\theta k_1)^\nu[g^{\mu\rho}k_3^2 - k_3^\mu k_3^\rho] \\ &+(\theta k_1)^\rho[g^{\mu\nu}k_2^2 - k_2^\mu k_2^\nu] + (\theta k_2)^\rho[g^{\mu\nu}k_1^2 - k_1^\mu k_1^\nu] \\ &+\theta^{\mu\alpha}(ak_1 + k_2 + k_3)_\alpha[g^{\nu\rho}(k_3k_2) - k_3^\nu k_2^\rho] \\ &+\theta^{\nu\alpha}(k_1 + ak_2 + k_3)_\alpha[g^{\mu\rho}(k_3k_1) - k_3^\mu k_1^\rho] \\ &+\theta^{\rho\alpha}(k_1 + k_2 + ak_3)_\alpha[g^{\mu\nu}(k_2k_1) - k_2^\mu k_1^\nu], \end{aligned}$$

and

$$\theta^{\mu\nu\rho} = \theta^{\mu\nu}\gamma^\rho + \theta^{\nu\rho}\gamma^\mu + \theta^{\rho\mu}\gamma^\nu.$$

Energy momentum conservation dictates that unitarity has to be satisfied [14]. The coupling of gauge boson to matter field preserves the gauge condition in energy momentum conservation. Therefore, under the production of each photon energy momentum k_1^μ and k_2^ν from the above couplings, we obtain that energy momentum is conserved when producing k_3^ρ lagged momentum. Momentum conservation in central mass frame is preserved on the coupling, but the energy asymmetry is not conserved in electric field ambience. The reason of the produced exotic energy is due to this coupling proportional to the coupling constant multiplying the central energy.

Following the discussion, if the process conatins triple gauge boson coupling by electric field. The exotic longitudinal state in charged matter current is naturally produced from the shifted charge ranged in its mass. The finial triple gauge boson coupling stores sufficient exotic energy transferring from gauge boson propagator to generate the non-physical state in the finial gauge boson luminosity.

If we choose the central mass frame, the collider phenomenon does not be changed in this frame, even if *Lorentz* invariance is violated. We choose $\theta^{0i}=0$, and set observer standing on the incident event. However, $k^1_\mu\theta^{\mu\nu}k^2_\nu$ is useless in the devotion without θ^{0i}. Moreover, numerical section we introduce how the first order $\theta_{\mu\nu}$ deformed term influences our results via background magnetic field, $B^i=\frac{1}{2}\epsilon^{ijk}\theta_{jk}$, couples to photon polarization. The finial results of forward-backward asymmetry is transparently indicated into parity violation effect.

III. $e^+e^-\to\gamma\gamma$ PHYSICS

We briefly review $e^+e^-\to\gamma\gamma$ process on noncommutative U(1) model[15]. The U(1) NCQED is a complete order $\theta_{\mu\nu}$ deformed field theory with containing even order $\theta_{\mu\nu}$ perturbation expansion. However, we read that the event number is like a sinuous function with parity is preserved in spite of containing triple photon coupling. It is well-known that noncommutative geometry is a nonlocal perturbative theory. It is seeming a phase transition in spacetime coordinates. This dramatic phenomenon is a complete background deformed effect.

The unusual commutation relation induces a triple gauge boson coupling on the electroweak model. Violating charge conservation, such as the couplings $\gamma-\gamma-\gamma$ and $Z-\gamma-\gamma$, is considered in amplitude[1]. U(1) gauge cannot produce parity violated phenomenon without considering $Chern-Simons$ term in the Lagrange or containing a non-equilibrium field in vacuum. There is no helicity violation generated in this group if no parity violation effects taking into account. The gauge field expansion are redefined as

$$\hat{A}_\mu = A_\mu - \frac{1}{2}\theta^{\alpha\beta}A_\alpha(\partial_\beta A_\mu + F_{\beta\mu}), \tag{8}$$

and its strength field

$$\hat{F}_{\mu\nu} = F_{\mu\nu} - \theta^{\alpha\beta}(A_\alpha\partial_\beta F_{\mu\nu} + F_{\mu\alpha}F_{\beta\nu}), \tag{9}$$

which it is a first order $\theta_{\mu\nu}$ expansion, where the background tensor is denoted by Eq.(1) and (2).

The polarization sum is revised to be the transition involved into noncommutative phase,

$$\sum_s \epsilon^{\star s}_\mu(k)\star\epsilon^s_\nu(k) = -\big(g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{n\cdot k} + \frac{n^2 k_\mu k_\nu}{(n\cdot k)^2}\big), \tag{10}$$

the noncommutative phase in front of the polarization sum gives us a lots clues of the coherent effect between photon polarization, but the induced $\theta_{\mu\nu}$ phase transition is useless on the collider process. In fact, although the U(1) model does not contain parity violated source without $Chern-Simons$, the odd order theta deformed term will deviate on the loop process, such as magnetic dipole moment and electric dipole moment [16].

Physically speaking, the background magnetic field induces a spin-magnetic effect, the term of charge violated coupling is simultaneously violating parity symmetry. Even in U(1) model, the perturbative expansion corrects all parity violated events on the odd order $\theta_{\mu\nu}$ deformation. The even order $\theta_{\mu\nu}$ deformations only contribute on the cross section magnitude. Therefore, it easily discovers that parity violated phenomenon on electron annihilation process is justified from the order of $s\times\Lambda_C^{-2}$ in series expansion,

$$\frac{d\sigma}{dzd\phi} = \frac{\alpha^2}{4s}\left[\frac{u}{t}+\frac{t}{u}-4\frac{u^2+t^2}{s^2}\sin^2(\frac{k_1\theta k_2}{2})\right], \tag{11}$$

which the last term is the same as the Compton process in exchanging p_2 and k_1, where $u=(p_1-k_2)^2$, $t=(p_1-k_1)^2$, and $s=(p_1+p_2)^2$. These processes are only contributed by the background electric field. It implies that the finial state photon does not interact with the background magnetic field, and its deviation is coming from the interaction between background electric field with electric charge.

We explore that electric field interaction with e^+ and e^- on the opposite influence by multiplying a constant b before of the imagine component in spinor vector polarization. If we choose the electric field direction perpendicular to the incoming incident, the event number is maximum distributioned. Expectedly, $\alpha_E=0$ does not contain ϕ dependent effect, because the preferred direction parallels to the incident axis. On the noncommutative electroweak model, due to the unitarity condition on the triple gauge boson coupling, we have to omit the background electric field automatically. However, in the U(1) case, the total cross section is proportional to the $\theta_{\mu\nu}$ second order term. If no preserced $\theta_{\mu\nu}$ odd order term in the result, therefore, no symmetry properties can be found.

Nonetheless, in noncommutative electroweak model, the term retains in the finial result. Intuitively, the process generates parity asymmetry effect. Following the diagrams we write down the square amplitude and photon polarization under the first order $\theta_{\mu\nu}$ deformation. Consider each photon polarization in

$$\epsilon_{1\mu}=(0,1,bi,0), \quad \epsilon_{2\mu}=(0,1,-bi,0), \tag{12}$$

and each incoming momentum and outgoing momen-

[1] Because C(γ) = C(Z) = - 1, but preserves CP symmetry

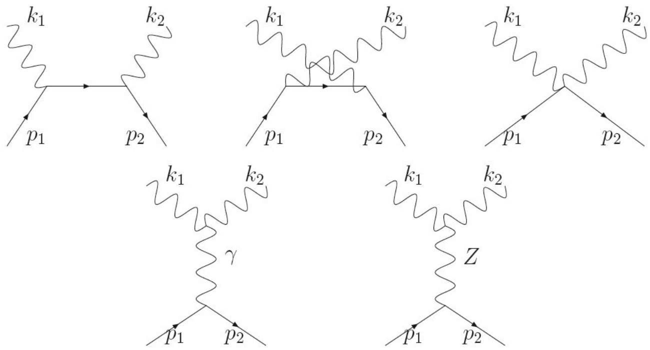

FIG. 1: The $e^+e^- \to \gamma\gamma$ diagrams

tum,

$$\begin{aligned} p_1^\mu &= (E, 0, 0, E), \quad p_2^\nu = (E, 0, 0, -E), \\ k_1^\mu &= E(1, \sin\theta\cos\phi, \sin\theta\sin\phi, 1), \\ k_2^\mu &= E(1, -\sin\theta\cos\phi, -\sin\theta\sin\phi, -1), \end{aligned} \tag{13}$$

in which b is + or - corresponding to right-handed and left-handed circle polarization with the incident working on the background

$$\vec{B} = \frac{1}{\Lambda_C}(\sin\alpha\sin\beta, \sin\alpha\sin\beta, \cos\alpha). \tag{14}$$

It is convenient to analyze the contribution of each different helicity.

We consider, θ^{ij}, space-space noncommutative deformed spacetime. The total amplitude splites into a zero term and a first order theta deformation,

$$\sigma_{tot} = \sigma_0 + \sigma_\theta \ (\texttt{theta first order term}),$$

the first part is the original commutative term and the second is the first order theta deformed term. It contributes to the total cross section with a free gauge freedom constant "a" and helicity constant "b". The renormalization condition requires the parameter "a" to be 3. The cross section zeroth and first order term are as follows,

$$\sigma_0 = \frac{\alpha^2}{4s}\left(\frac{t}{u} + \frac{u}{t}\right), \tag{15}$$

$$\sigma_\theta = \frac{\alpha^2}{4s}\mathfrak{Re}\big[\sigma_1 + (a+1)\sigma_2\big], \tag{16}$$

where

$$\sigma_1 = -\frac{i}{2}(\epsilon_1\theta\epsilon_2)\left[\frac{s^2 b\triangle}{2} + sz(s\Box - 1)\right], \tag{17}$$

$$\sigma_2 = \frac{i}{2}s^{\frac{3}{2}}\triangle\sqrt{\frac{1-z^2}{2}} \left[\frac{(\epsilon_1\theta k_1)(i\sin\phi - b\cos\phi)}{1+z} + \frac{(\epsilon_2\theta k_1)(i\sin\phi + b\cos\phi)}{1-z}\right], \tag{18}$$

and

$$\begin{aligned} \triangle &= \frac{2K_{Z\gamma\gamma}C_A}{s - m_Z^2}, \\ \Box &= -\frac{2K_{\gamma\gamma\gamma}\sin 2\theta_W}{s} - \frac{2C_V K_{Z\gamma\gamma}}{s - m_Z^2}, \\ C_{V,f_L} &= T_{3,f_L} - 2Q_f\sin^2\theta_w, \\ C_{A,f_L} &= T_{3,f_L}. \end{aligned}$$

The total decay rate is contributed from complete theta second order modification. Hence, no asymmetry phenomenon can be generated. Under the rotation of the background field direction, the total decay rate is symmetrically rotated with the angular momentum correlation between original axis and background unique direction. The $Z^0 \to \gamma\gamma$ decay [11] is completely forbidden by angular conservation and bosonic distribution.

IV. NUMERICAL RESULT

In the numerical analysis, the influence of the background field direction dominates the total cross section and differential cross section of each helicity state. Each photon helicity interacts with the background magnetic field in the opposite distribution. The asymmetry effects in the finial helicity state are mutually canceled on the unpolarized cross section. Moreover, concentrating on the result of the $\theta_{\mu\nu}$ deformed term, Eq.(16, 17, 18), we show that the Z_0 gauge boson mediator is almost completely violating parity asymmetry. The contribution of the massless gauge boson on each helicity state does not cause rapid changes. The cross section is minutely varied, but, its behavior is dramatically changed on the scattering process. Because the Z^0 gauge boson is working on a non-abelian gauge and coupling to opposite helicity current by different distribution with the mass approaching to 0.1 TeV.

In Fig.(2), the contribution of the Z^0 gauge boson process dominates the total cross section, and the results compare with the unpolarized beam in setting $K_{Z\gamma\gamma}$ = - 0.2. The Z^0 gauge mediator produce a slight shift, but the photon sector will not be changed. In the $SU_L(2) \otimes U_Y(1)$ model, photon is a gauge boson coupling to each helicity current by the same phenomenon. Z^0 gauge boson induces a different distribution in the left-handed and right-handed currents. Therefore, on the Left-Right symmetry model, $SU_C(3) \otimes SU_R(2) \otimes SU_L(2) \otimes U_Y(1)$, the asymmetry effect wishes to disappear on the unpolarized Z^0 channel. Due to unitary constraint on the gauge sector, it should be conserved on the SU(N) group. Hence, Seiberg-Witten map cannot give the other clues to allure us to do the work in extending gauge sector from choosing larger $\theta_{\mu\nu}$ expansion.

The total cross section cannot be corrected with setting $K_{\gamma\gamma\gamma} = 0$ in Fig.(3), since photon is a complete U(1) gauge boson. In high energy level, the main distribution presents along the z-axis. As to the ϕ-axis, the influence of the spin-magnetic interaction is very little

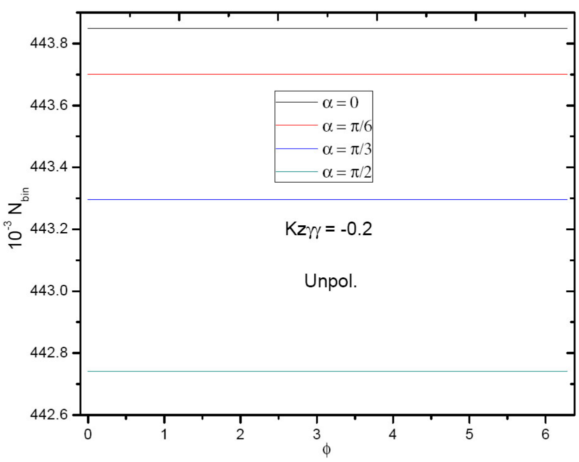

FIG. 2: The coupling constant $K_{Z\gamma\gamma}$ = -0.2, where E_{CE} = 800 GeV, Λ_C = 1TeV. As the result from Z^0 gauge boson couples to the matter current by the different contribution, therefore it will contribute on the unpolarized cross section. If $K_{Z\gamma\gamma} = 0$ the number of event approaches to QED result 442.74.

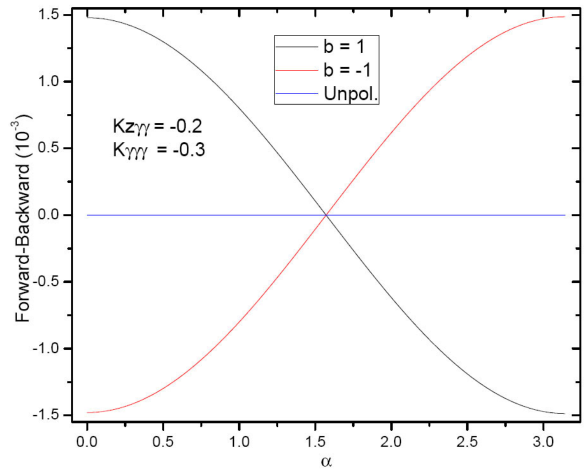

FIG. 4: The forward-backward asymmetry is mainly affected by the cube vertex diagram, the Z^0 mediator gauge boson contributed effect is actually very small. The coupling constants $K_{Z\gamma\gamma}$ = -0.2, $K_{\gamma\gamma\gamma}$ = -0.3, and cnetral energy E_{CM} = 800 GeV at Λ_C = 1TeV.

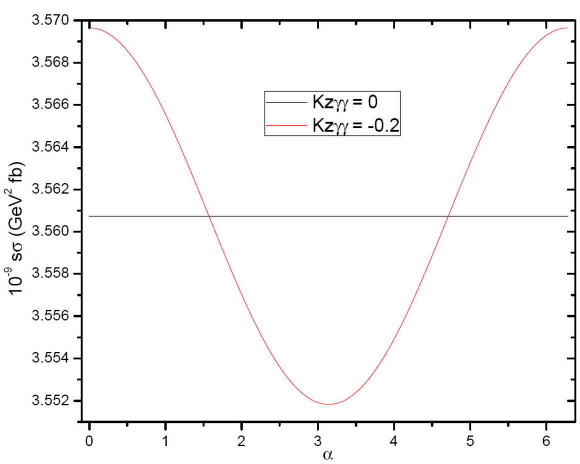

FIG. 3: The coupling constant $K_{Z\gamma\gamma}$ = 0 and -0.2, and the central energy E_{CM} = 800 GeV at Λ_C = 1TeV scale. As $K_{Z\gamma\gamma}$ = 0, that will be as same as QED result, 3.561 unit. In the polarized helicial state, the contribution of b = 1, and b = -1 on the background field direction along the z-axis are the same.

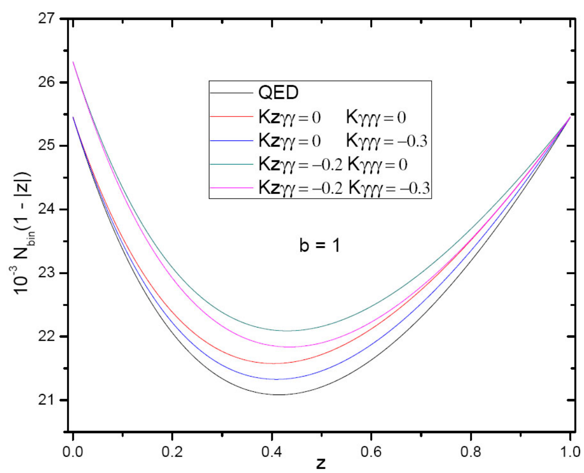

FIG. 5: The coupling constant $K_{Z\gamma\gamma}$ = -0.2 and 0, the $\alpha = \frac{\pi}{3}$ and E_{CE} = 800GeV at Λ_C = 1TeV. The black dashline is the original QED prediction.

influenced. Visibly, the diagram, Fig.(3), is a perfect symmetry on the limit point $\alpha = \pi$. It is a result in assuming two observers stand on the either sides of the event point. They cannot get the same result as detecting the total cross section of each photon helicity. The order of difference quality is associated with the squared inverse of the Λ_c parameter. Throughout the F-B asymmetry discussion, we set the parameter Λ_c to 1 TeV, and the central energy is assumed to be 800 GeV.

The main idea of parity violation, Fig.(4)(5), is a spin-magnetic field interaction. Which is contributed on the difference energy distributions on the opposite sides of the event point. If spin orientation is parallel to the background magnetic field, the energy distribution is maximally contributed. In contrast, energy is diminished if the direction between spin and background magnetic field is opposed with the angle depends on the z-axis and the background preference. It is the reason why we can get parity violation phenomena. The term of $\vec{S}\times\vec{B}$ gives us a different physics viewpoint to investigate the process. This term clearly indicates spin cannot be perpendicular to the background magnetic field. The difference of varying parity asymmetry associates with the strength of background magnetic field.

Another, observable evidences, Fig.(6), are the quantity of event number as to the z variable, z = $\cos\theta$. The helicity is contributed by the parity asymmetry effects on finial result. We consider helicity state $b = 1$ and discuss, however, that another helicity state b = -1 is shifted on the opposite side. If we set the $K_{\gamma\gamma\gamma}$ coupling equals to zero then the signal is similarly unchanged, because Z^0 boson is heavier than photon. Photon gauge boson contributes to the finial result has perceived more

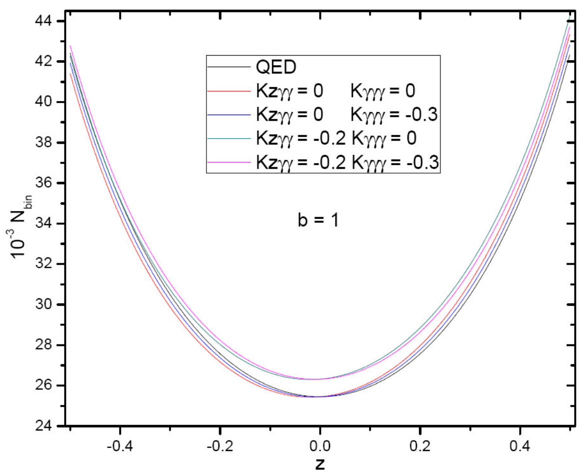

FIG. 6: The coupling $K_{Z\gamma\gamma}$ = -0.2, 0 and $K_{\gamma\gamma\gamma}$ = -0.3, 0, where $\alpha = \frac{\pi}{3}$, $E_{CE} = 800$GeV, and $\Lambda_C = 1$TeV. Comparing to the QED result, the coupling $K_{\gamma\gamma\gamma}$ will be dominant in influency the slight shift effects.

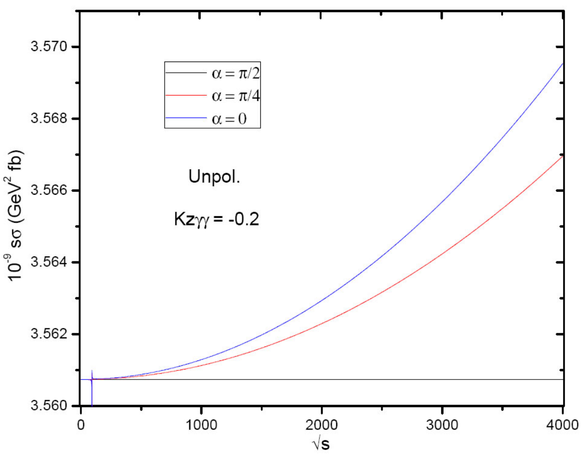

FIG. 7: The Unpolarized cross section, we set the coupling constant $K_{Z\gamma\gamma}$ = -0.2, where the Λ_C is set to 5000 GeV. The energy spectrum is increasd by the spin-magnetic production effect, gauge boson polarization coupling to the background magnetic field on the event point will rearrange the distribution of energy production.

than massive Z^0. The shift devotes on the photon spin interacts with background magnetic field, and the axis is perpendicular to the direction. The external distribution is perpendicular to the axis of the magnetic field direction, because the effective term of $\vec{S} \times \vec{B}$ generates a partial vector paralleling to the plane.

The energy spectrum, Fig.(7), in ranging the central energy, the $\theta_{\mu\nu}$ expansion plays an important role on varying the associated angle. The $\alpha = 0$ generates a distribution at high energy level because the polarization of the total cross section on the event point is parallel to the beam axis. However, we have mentioned that if the polarization is parallel to the magnetic field, thus, the result obtains the maximum energy distribution function. Such as the concept of quantum mechanics, the energy spectrum is decided by the eigenvalues of the global system. Therefore, $\alpha = 0$, on the event point, photon gain a maximum energy distribution on the collision process. Its luminosity contains tiny difference as to the movement of earth.

V. CONCLUSION

We have briefly introduced how the background magnetic field influences the electron annihilation to two photons process. A strong magnetic field induces an interesting effect under the exotic massive gauge boson Z^0 and massless photon. However, parity violation is observed on the further high energy level. CP symmetry is still conserved on the triple photon and Z^0 gauge boson coupling, due to these couplings violate charge and parity asymmetry. Thus, the exotic term in the action deformed by $\theta_{\mu\nu}$ expansion cannot induce the CPV effects. However , the energy spectrum, due to particle spin, interacts with magnetic field to generate a difference energy distribution on the opposite sides around the event point. The energy distribution dominantly induces the parity asymmetry on the observer stage. Therefore, this process is a contribution of a better understanding of further probing background field situation.

Acknowledgments

We will thank Chao Qiang Geng, Xiao Gang He, and J. N. Ng for useful discuss and the National Science Council of R.O.C. under contact : NSC-95-2112-M-007-059-MY3 and National Tsing Hua University under contact : 97N2309F1.

[1] N. Seiberg and E. Witten, JHEP **9909**, 032 (1999) [arXiv:hep-th/9908142]; N. Seiberg, L. Susskind and N. Toumbas, JHEP **0006**, 021 (2000) [arXiv:hep-th/0005040]; A. Abouelsaood, C. G. . Callan, C. R. Nappi and S. A. Yost, Nucl. Phys. B **280**, 599 (1987).

[2] S. Minwalla, M. Van Raamsdonk and N. Seiberg, JHEP **0002**, 020 (2000) [arXiv:hep-th/9912072]; A. Micu and M. M. Sheikh Jabbari, JHEP **0101**, 025 (2001) [arXiv:hep-th/0008057]; I. Y. Aref'eva, D. M. Belov, A. S. Koshelev and O. A. Rytchkov, Nucl. Phys. Proc. Suppl. **102**, 11 (2001) [arXiv:hep-th/0003176]; L. Griguolo and M. Pietroni, JHEP **0105**, 032 (2001) [arXiv:hep-th/0104217]; T. R. Govindarajan, S. Kurkcuoglu and M. Panero, Mod. Phys. Lett. A **21**, 1851 (2006) [arXiv:hep-th/0604061].

[3] M. Panero, JHEP **0705**, 082 (2007) [arXiv:hep-th/0608202].

[4] B. Jurco, S. Schraml, P. Schupp and J. Wess, Eur. Phys. J. C **17**, 521 (2000) [arXiv:hep-th/0006246]; J. Madore, S. Schraml, P. Schupp and J. Wess, Eur. Phys. J. C **16**, 161 (2000) [arXiv:hep-th/0001203]; L. Bonora, M. Schnabl, M. M. Sheikh-Jabbari and A. Tomasiello, Nucl. Phys. B **589**, 461 (2000) [arXiv:hep-th/0006091]; K. Matsubara, Phys. Lett. B **482**, 417 (2000) [arXiv:hep-th/0003294].

[5] M. Chaichian, P. Presnajder, M. M. Sheikh-Jabbari and A. Tureanu, Phys. Lett. B **526**, 132 (2002) [arXiv:hep-th/0107037].

[6] R. Wulkenhaar, Lect. Notes Phys. **596**, 313 (2002) [arXiv:hep-th/9912221]; P. Schupp, [arXiv:hep-th/0111038].

[7] C. P. Martin, Nucl. Phys. B **652**, 72 (2003) [arXiv:hep-th/0211164].

[8] J. Zahn, Phys. Rev. D **73**, 105005 (2006) [arXiv:hep-th/0603231]; M. Chaichian, P. P. Kulish, K. Nishijima and A. Tureanu, Phys. Lett. B **604**, 98 (2004) [arXiv:hep-th/0408069].

[9] J. C. Wallet, J. Phys. Conf. Ser. **103**, 012007 (2008) arXiv:0708.2471 [hep-th].

[10] X. Calmet, B. Jurco, P. Schupp, J. Wess and M. Wohlgenannt, Eur. Phys. J. C **23**, 363 (2002) [arXiv:hep-ph/0111115]; B. Melic, K. Passek-Kumericki, J. Trampetic, P. Schupp and M. Wohlgenannt, Eur. Phys. J. C **42**, 483 (2005) [arXiv:hep-ph/0502249].

[11] G. Duplancic, P. Schupp and J. Trampetic, Eur. Phys. J. C **32**, 141 (2003) [arXiv:hep-ph/0309138]; J. Trampetic, SFIN A **1**, 379 (2007) [arXiv:0704.0559 [hep-ph]]; M. Buric, D. Latas, V. Radovanovic and J. Trampetic, Phys. Rev. D **77**, 045031 (2008) [arXiv:0711.0887 [hep-th]]; A. P. Balachandran and S. G. Jo, Int. J. Mod. Phys. A **22**, 6133 (2007) arXiv:0704.0921 [hep-th]; W. Behr, N. G. Deshpande, G. Duplancic, P. Schupp, J. Trampetic and J. Wess, Eur. Phys. J. C **29**, 441 (2003) [arXiv:hep-ph/0202121].

[12] H. S. Snyder, Phys. Rev. **71**, 38 (1947).

[13] P. P. Kulish, [arXiv:hep-th/0606056]; V. Rivasseau, arXiv:0705.0705 [hep-th]; R. J. Szabo, AIP Conf. Proc. **917**, 146 (2007) [arXiv:hep-th/0701224].

[14] T. Ohl, R. Ruckl and J. Zeiner, Nucl. Phys. B **676**, 229 (2004) [arXiv:hep-th/0309021]; J. Gomis and T. Mehen, Nucl. Phys. B **591**, 265 (2000) [arXiv:hep-th/0005129]; M. Chaichian, A. Demichev, P. Presnajder and A. Tureanu, Eur. Phys. J. C **20**, 767 (2001) [arXiv:hep-th/0007156].

[15] S. Baek, D. K. Ghosh, X. G. He and W. Y. P. Hwang, Phys. Rev. D **64**, 056001 (2001) [arXiv:hep-ph/0103068]; J. L. Hewett, F. J. Petriello and T. G. Rizzo, Phys. Rev. D **64**, 075012 (2001) [arXiv:hep-ph/0010354]. P. Mathews, Phys. Rev. D **63**, 075007 (2001) [arXiv:hep-ph/0011332]; T. G. Rizzo, Int. J. Mod. Phys. A **18**, 2797 (2003) [arXiv:hep-ph/0203240].

[16] I. F. Riad and M. M. Sheikh-Jabbari, JHEP **0008**, 045 (2000) [arXiv:hep-th/0008132]; N. Kersting, Phys. Lett. B **527**, 115 (2002) [arXiv:hep-ph/0109224].